

MATHCOUNTS®

2019

■ Mock Chapter Competition ■ Sprint Round Problems 1–30

HONOR PLEDGE

I pledge to uphold the highest principles of honesty and integrity as a Mathlete®. I will neither give nor accept unauthorized assistance of any kind. I will not copy another's work and submit it as my own. I understand that any competitor found to be in violation of this honor pledge is subject to disqualification.

Signature _____ Date _____

Printed Name _____

School _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of 30 problems. You will have 40 minutes to complete all the problems. You are not allowed to use calculators, books or other aids during this round. If you are wearing a calculator wrist watch, please give it to your proctor now. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the left-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

In each written round of the competition, the required unit for the answer is included in the answer blank. The plural form of the unit is always used, even if the answer appears to require the singular form of the unit. The unit provided in the answer blank is the only form of the answer that will be accepted.

Total Correct	Scorer's Initials

Raytheon

2019 MATHCOUNTS
National Competition Sponsor

NATIONAL SPONSORS

Raytheon Company
Northrop Grumman Foundation
U.S. Department of Defense
National Society of Professional Engineers
CNA Insurance
Phillips 66
Texas Instruments Incorporated
3Mgives
Art of Problem Solving
NextThought

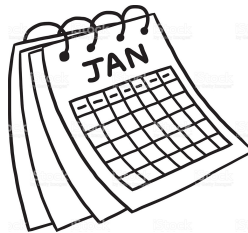
FOUNDING SPONSORS: National Society of Professional Engineers, National Council of Teachers of Mathematics and CNA Foundation

1. _____ What is the value resulted when the number fifty-four is subtracted from the number two thousand?

2. _____ What is the sum of the two integers closest to the square root of 30?

3. _____ °F Absolute zero is the lowest possible temperature, which is approximately -459.7 °F. The boiling point of water is 212 °F. What is the sum of the boiling point of water and absolute zero? Express your answer as a decimal to the nearest tenth.

4. _____ problems Annie solves 930 math problems in the month of January. How many problems, on average, does she solve each day in that particular month?



5. _____ If $5a + 1 = 15$, then what is the value of $7a$? Express your answer as a common fraction.

6. _____ hours Darla currently has 17 robots, and she builds 5 more robots each hour. Mark currently has 71 robots, and he destroys 2 of his robots each hour. In how many hours will Darla and Mark have the same amount of robots?

7. _____ Roland received the following grades on his report card: 100, 97, 99, 91, 98, 96, 94, 99. What is the sum of the mean, median, and mode of the grades on his report card? Express your answer as a decimal to the nearest hundredth.

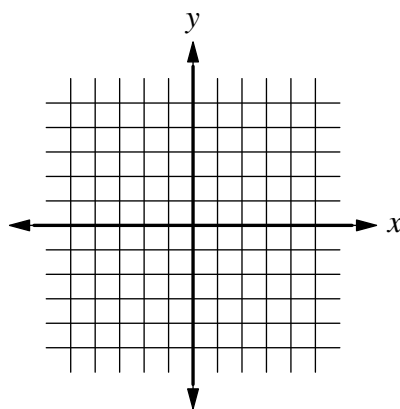
8. \$ _____ Pranav has a certain amount of money at the beginning of the day. He spends a quarter of the money on groceries. He spends two-thirds of the remaining money on gasoline, and then ends up with exactly \$35. How much money did Pranav have at the beginning of the day?

9. _____ percent The table shows how many students at Westside Middle School are in a particular club. If each student that is in a club belongs only to that club, then what percent of the students at Westside Middle School are in sports club?

CLUBS AT WESTSIDE MIDDLE SCHOOL	
Club	# Students
Math	23
Science	44
Spelling Bee	13
Sports	36
Book	19
None	135

10. _____ If the sum of the square roots of two numbers is equal to the square root of the sum of the two numbers, what is the product of the two numbers?

11. _____ If the number $a89b$ is divisible by 3, for a and b are digits and $a > 0$, what is the sum of all the possible values of $a + b$?
12. ordered pairs How many ordered pairs of integers (x, y) satisfy the equation $\frac{6}{x} + \frac{4}{y} = 1$?
13. _____ When a positive integer n is divided by 7, the remainder is 5. What is the remainder when $6n$ is divided by 7?
14. _____ If -1 and 6 are the roots to the equation $3x^2 - mx + n = 0$, what is $m + n$?
15. units² What is the area of a polygon having vertices with coordinates $(0, 3)$, $(5, 0)$, $(0, -4)$, and $(-1, 0)$?



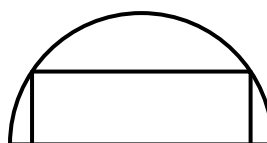
16. _____ If the sum of two numbers is 6 and four times the smaller number subtracted from three times the larger number is 4, what is the positive difference between the squares of the both numbers?
17. _____ Seven positive integers exist such that the mean of these integers is 8, and the median is 7. What is the greatest possible integer in this set of integers?
18. _____ If $f(x) = x^2 + 5x + 4$ and $g(x) = x^2 + 3x - 4$, then what is $\frac{g(f(3))}{f(g(3))}$? Express your answer as a common fraction.
19. _____ cars The local car dealership owns only blue and red cars. The ratio of the number of blue cars to red cars at the dealership is 5:3. A shipment of 24 blue cars and 3 red cars is delivered to the dealership, causing the ratio of blue cars to red cars to be 2:1. How many total cars are there after the shipment?
20. _____ values How many integer values of x satisfy the equation $\left| \frac{1}{2}x + 4 \right| < 6$?

21. _____ Let a , b , and c all be numbers such that the arithmetic mean of a and b is 3, the arithmetic mean of b and c is 9, and the arithmetic mean of a and c is 12. What is the arithmetic mean of a , b , and c ?

22. _____ What is the greatest integer value of n such that $n!$ is less than $(5n)^2$?

23. _____ ways How many ways are there to give sixteen gifts to four children, if each child must receive at least two gifts?

24. _____ A rectangle is inscribed inside of a semicircle. The ratio of the shorter side of the rectangle to its longer side is 1:3. What is the ratio of the area of the semicircle to that of the rectangle? Express your answer as a common fraction in terms of π .

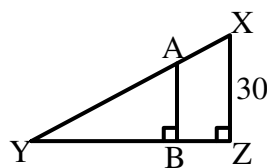


25. _____ If x is an integer, what is the sum of the values of x that make $(x^2 - 4x + 5)(x^2 + 6x + 9)$ a prime number?

26. _____ Ana and Bobbie each arrive at a party at a random time between 3:00 and 7:00, stay for 5 minutes and then leave. What is the probability that Ana and Bobbie see each other at the party? Express your answer as a common fraction.

27. _____ hours Zara has a house with 4 walls and a ceiling. Alex can paint a wall of Zara's house in 2 hours, and he can paint the ceiling in 3 hours. Beth can paint a wall in $1\frac{1}{2}$ hours, and paint the ceiling in $2\frac{1}{2}$ hours. How long would it take for Alex and Beth to paint Zara's whole house if they both start painting the walls at the same time and then paint the roof when the walls are fully painted? Express your answer as a common fraction.

28. _____ units² Right triangle XYZ has $XZ = 30$, as shown below. Points A and B lie on sides XY and YZ, respectively. If $AX = 17$ and $BZ = 15$, what is the area of triangle XYZ?



29. _____ points There are exactly 5 lattice points contained within the graph of $x^2 + y^2 \leq 1$. How many lattice points are contained within the graph of $4(x - 5)^2 + 4(y + 4)^2 \leq 400$?

30. _____ The dartboard shown has a radius of 8 inches and is labeled with point values. Each of the concentric circles has a radius two inches less than the next larger circle. If Bart throws three darts randomly at the target, what is the probability that he earns at least twenty points? Express your answer as a common fraction.

